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Aggregate size distributions in migration driven growth models

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Abstract. The kinetics of aggregate growth through reversible migrations between any two aggregates is studied. We propose a simple model with the symmetrical migration rate kernel $K(k;j) \propto (kj)^{\upsilon}$ at which the monomers migrate from the aggregates of size k to those of size j. The results show that for the $\upsilon \leq 3/2$ case, the aggregate size distribution approaches a conventional scaling form; moreover, the typical aggregate size grows as $t^{1/(3-2\upsilon)}$ in the $\upsilon < 3/2$ case and as $\exp(C_1 t)$ in the $\upsilon = 3/2$ case. We also investigate another simple model with the asymmetrical rate kernel $K(k;j) \propto k^{\mu} j^{\nu}$ $(\mu \neq \nu)$, which exhibits some scaling properties quite different from the symmetrical one. The aggregate size distribution satisfies the conventional scaling form only in the case of $\mu < \nu$ and $\mu + \nu < 2$, and the typical aggregate size grows as $t^{2-\mu-\nu}$.

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1 Introduction

Recently, much interest has been devoted to investigating the kinetics of irreversible aggregation processes in which the aggregates grow through the binary coalescence mechanism, and considerable understanding of such aggregation processes has been achieved [1–8]. The general binary mechanism of irreversible aggregation is described by the reaction scheme $A_i + A_j \longrightarrow A_{i+j}$, where A_i denotes an aggregate consisting of i monomers. This is, the aggregates A_i and A_j can bond spontaneously and result in a larger aggregate A_{i+j}. However, another important evaporation/condensation mechanism also occurs frequently in physics and in social science [9–14]. It is preferential evaporation from smaller aggregates and preferential condensation onto larger aggregates. In social science, the growth of cities is driven by migration from the small cities to the large ones [11], and the wealth redistribution of the individuals may be caused by the "asset migration" from the poor to the rich individuals in economical interactions [15]. In order to seek the universal aggregation mechanism for the evolution of city populations, Leyvraz and Redner proposed a migration-driven aggregation model and established a scaling theory for aggregate growth [16]. In these models, the irreversible migration scheme is $A_k + A_l \longrightarrow A_{k-1} + A_{l+1}$ $(k \leq l)$. That is, a monomer leaves a smaller aggregate of size k and joins a larger one of size l. The results exhibited that the kinetics of these processes falls in a scaling regime different from conventional aggregation processes. More generally, the migration direction may not depend on the relative sizes of the two aggregates. In our previous work [17], we investigated the kinetics of the migration-driven aggregation model with a special migration rate kernel, in which migrations go from the larger aggregates to the smaller ones as well as from the smaller to the larger, and it was found that the evolution behavior of this system obeys a conventional scaling law different from the above mentioned preferential migration model in reference [16]. These indicate that the migration-driven aggregation mechanism gives rise to rich kinetic scaling behaviors of aggregate growth.

In this paper, we investigate simple migration-driven aggregation models without any bias of the migration direction. This is, the system evolves according to a non-directional migration scheme, $A_k + A_l \overset{K(k;l)}{\longrightarrow} A_{k-1} + A_{l+1}$, where $K(k;l)$ denotes the rate at which the A_k aggregate loses a monomer to the A_l aggregate independent of their relative sizes. We shall study our model on the basis of the mean-field assumption. The system is assumed to be of spatial homogeneity, so that the fluctuations in the densities of the reactants are negligible and the aggregates are considered to be homogeneously distributed in the space throughout the process. However, the mean-filed assumption typically applies to the case in which the spatial dimension d of the system is equal to or greater than a critical dimension d_c [18]. For the $d < d_c$ case, the fluctuations in the reactant densities cannot be ignored and the mean-field approach is thus invalid. Some more sophisticated methods are required to describe the kinetics of such low-dimension systems [19,20].

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It is believed that the migration-driven aggregation model may mimic many phenomena such as the distribution of city populations and the wealth distribution in social science. We also believe that the migration systems are of interest in studying the scaling properties of their evolution behaviors. Based on the mean-field rate equations of the migration processes, we determine the asymptotic solutions of the aggregate size distributions in two distinct systems. The results show that the aggregate size distribution approaches the conventional scaling form in the long-time limit. However, the scaling properties of the system with a symmetrical rate kernel are quite different from those of the system with an asymmetrical rate kernel.

The paper is organized as follows. In Section 2, we propose a reversible migration-driven aggregation model with a symmetrical rate kernel and investigate the mean-field rate equations to obtain the aggregate size distributions. In Section 3, we then study the kinetic behavior of the migration-driven aggregate growth system with an asymmetrical rate kernel. A brief summary is given in Section 4.

2 The model with a symmetrical migration rate kernel

In the mean-field limit, the theoretical approach to investigate the kinetics of the aggregation models can be based on the rate equation, which assumes that the reaction proceeds at a rate proportional to the concentration of each reactant. Let $a_k(t)$ be the concentration of the aggregates A_k of size k at time t. Then the rate equation for our system reads

$$\frac{da_k}{dt} = \frac{1}{2}\sum_{i,j=1}^{\infty} K(i;j)a_i a_j(\delta_{i,k+1}+\delta_{j,k-1}-\delta_{i,k}-\delta_{j,k}), \quad (1)$$

where we impose the boundary condition $a_0(t) \equiv 0$. From equation (1) one easily finds that the total mass of the aggregates is conserved, namely, $\dot{M}_1(t) = \sum_{k=1}^{\infty} k\dot{a}_k(t) \equiv 0$. This indicates that the total size of the aggregates is naturally conserved by the dynamics of this system without any consumption of the aggregates.

We consider a simple model with a symmetrical rate kernel $K(k;j) = 2I(kj)^{\upsilon}$ (I is a constant). That is, the rate at which the monomers migrate from the A_k aggregate to the A_j aggregate is proportional to the values k^{υ} and j^{υ}, where υ is the migration rate index which may interpret the degree of "richness" in the population of a city. When the value of υ increases, the aggregate (city) becomes much generous in emigration and much greedy in immigration. This restriction of the rate kernel follows the spirit in which the model of random asset exchange is formulated [15]. The resulting model can be explicitly solved and thus provides a helpful understanding of the reversible migration mechanism, although it also has little realistic basis. On the other hand, it is natural that the overall migration rate varies as a power law in some scale factor [16]. Then equation (1) is rewritten as

$$\frac{da_k}{dt} = IM_{\upsilon}[(k+1)^{\upsilon}a_{k+1} + (k-1)^{\upsilon}a_{k-1} - 2k^{\upsilon}a_k], \quad (2)$$

with the shorthand notation $M_{\upsilon}(t) \equiv \sum_{j=1}^{\infty} j^{\upsilon}a_j(t)$. This model is similar to the exchange driven growth model with the product kernel $K(i;j) = (ij)^{\lambda}$ which was carefully studied in reference [21]. Here we shall investigate the solution of equation (2) by employing a different technique that is also useful for the following model with an asymmetrical kernel.

We assume that there only exist monomer aggregates at $t = 0$ and their concentration is equal to unity. So the initial condition of this system is $a_k(0) = \delta_{k1}$, $k = 1, 2, \ldots$ Obviously, we have $M_1(t) \equiv 1$. We then consider several simple cases with integral index υ which can be explicitly solved and help to predict the evolution properties of $a_k(t)$ in general cases.

When $\upsilon = 1$, equation (2) reduces to

$$\frac{da_k}{dt} = I[(k+1)a_{k+1} + (k-1)a_{k-1} - 2ka_k]. \quad (3)$$

Under the above monodisperse initial condition, equation (3) can be solved with the help of ansatz [18]

$$a_k(t) = A(t)[a(t)]^{k-1}. \quad (4)$$

Substituting equation (4) into equation (3), we can transform the rate equation (3) into two differential equations as follows:

$$\frac{da}{dt} = I(1-a)^2, \quad \frac{dA}{dt} = -2IA(1-a), \quad (5)$$

with the corresponding initial condition $a(t=0) = 0$ and $A(t=0) = 1$. From equations (5) one can then determine the exact solutions

$$a(t) = 1 - (It+1)^{-1}, \quad A(t) = (It+1)^{-2}. \quad (6)$$

Thus we obtain the exact solution of $a_k(t)$,

$$a_k(t) = A(t)[a(t)]^{k-1} = (It+1)^{-2}[1-(It+1)^{-1}]^{k-1}. \quad (7)$$

It is found that in the scaling region of $k \gg 1$ and $t \gg 1$, the aggregate size distribution (7) can be approximately rewritten as the well-known Smoluchovski scaling form (see, e.g., Refs. [18,22,23])

$$a_k(t) \simeq (It)^{-2}\exp(-x), \quad x = k(It)^{-1}. \quad (8)$$

The results show that in this case, all the aggregate size distribution $a_k(t)$ continuously goes to zero at $t \to \infty$.

When $\upsilon = 0$, equation (2) is rewritten as

$$\frac{da_k}{dt} = IM_0(t)[a_{k+1} + a_{k-1} - 2a_k]. \quad (9)$$

Equation (9) is similar to the governing rate equation for the random asset exchange model in reference [15]. In the long-time limit, by using the similar method in reference [15] we determine the asymptotic solution of the aggregate size distribution

$$a_k(t) \simeq \frac{k}{3It} \exp\left[-\left(\frac{\pi}{144}\right)^{1/3} \frac{k^2}{(It)^{2/3}}\right], \qquad (10)$$

which approaches the conventional scaling form [5]

$$a_k(t) \simeq k^{-\tau} t^{-w} \Phi[k/S(t)], \quad S(t) \propto t^z. \qquad (11)$$

Here $S(t)$ denotes the typical aggregate size of the system, which plays a role analogous to that of the correlation length in critical phenomena, and the scaling function $\Phi(x) \simeq 1$ for $x \ll 1$ and $\Phi(x) \simeq 0$ for $x \gg 1$. In this case, the scaling function takes the stretched exponential expression $\Phi(x) = \exp(-x^2)$ and the typical aggregate size grows as $t^{1/3}$. The governing exponents for this case are $\tau = -1$, $w = 1$, and $z = 1/3$. Moreover, each $a_k(t)$ continuously decreases to zero at the end.

Now we investigate the general cases. Under the monodisperse initial condition, we analyze the asymptotic solutions of $a_k(t)$ in the $\upsilon = 0, 1$ cases and, therefore, make the conventional scaling ansatz (11) for the long-time behavior of $a_k(t)$. Since our system obeys the mass conservation law, one can easily find the following exponent relation in the scaling form (11):

$$w = (2 - \tau)z. \qquad (12)$$

Equation (12) implies that for the system in which the total mass is conserved, the scaling ansatz (11) is valid only in the case of $\tau < 2$. For $\tau \geq 2$, this can never be the case unless the system gels because the total density of aggregates in the nongelling system always decreases with time. Here we only consider the nongelling case, i.e., $\tau < 2$. Inserting equation (12) into equation (11), we rewrite the scaling ansatz as follows:

$$a_k(t) \simeq k^{-\tau} [S(t)]^{\tau-2} \Phi[k/S(t)], \quad S(t) \propto t^z. \qquad (13)$$

Then the problem reduces to finding the index τ, the typical aggregate size $S(t)$, and the scaling function $\Phi(x)$.

We first determine the typical aggregate size $S(t)$. By making use of equation (13) one can find $M_2(t) \simeq S(t)$. Multiplying equation (2) with k^2 and summing them up over all k, we obtain

$$\frac{dM_2}{dt} = 2IM_\upsilon^2. \qquad (14)$$

It is not difficult to find that for the scaling function $\Phi(x)$ defined in equation (11), $\sum_{j=1}^{\infty} j^\omega \Phi[j/S(t)]$ is approximately equal to $C_{1\omega}[S(t)]^{\omega+1}$ for $\omega > -1$, $C_{2\omega} \ln[S(t)]$ for $\omega = -1$, or $C_{3\omega}$ for $\omega < -1$ (here $C_{1\omega}$, $C_{2\omega}$, and $C_{3\omega}$ are integration constants dependent on the value of the index ω). By substituting equation (13) into equation (14) we obtain

$$\frac{dS}{dt} \simeq \begin{cases} S^{2\upsilon-2} & \text{if } \tau < \upsilon + 1, \\ S^{2\tau-4}(\ln S)^2 & \text{if } \tau = \upsilon + 1, \\ S^{2\tau-4} & \text{if } \tau > \upsilon + 1. \end{cases} \qquad (15)$$

On the other hand, we sum up equation (2) and then obtain the evolution behavior of $M_0(t)$ as follows:

$$\frac{dM_0}{dt} = -Ia_1 M_\upsilon. \qquad (16)$$

It is found that $M_0(t) \simeq 1/S(t)$ for $\tau < 1$ and $M_0(t) \simeq \ln S(t)/S(t)$ for $\tau = 1$ and $M_0(t) \simeq [S(t)]^{\tau-2}$ for $\tau > 1$. Inserting equation (13) into equation (16), one can easily obtain another set of evolution behaviors of $\dot{S}(t)$ in different cases. In the case of $\tau \geq \upsilon + 1$, it follows from equation (16) that $\dot{S}(t) \propto S^{2\tau-2}$ for $\tau \leq 1$ and $\dot{S}(t) \propto S^{\tau-1}$ for $\tau > 1$, which are quite different from those corresponding expressions in equation (15). So we know that for this system, the asymptotic solution (15) of $\dot{S}(t)$ is only valid in the case of $\tau < \upsilon + 1$. Moreover, from equation (16) we can find that in the $\tau < \upsilon + 1$ case,

$$\frac{dS}{dt} \simeq \begin{cases} S^{\upsilon+\tau-1} & \text{if } \tau < 1, \\ S^\upsilon \ln S(t) & \text{if } \tau = 1, \\ S^\upsilon & \text{if } \tau > 1. \end{cases} \qquad (17)$$

Making a comparison between equations (15) and (17) shows that $\tau = \upsilon - 1$.

In the $\upsilon > 3/2$ case, from equation (15) we find that the typical size $S(t)$ grows only at finite time $t < t_c$ (t_c is the critical time of the gel point) and the scaling form of equation (11) is thus invalid. Moreover, gelation is complete in the case of $2 > \upsilon > 3/2$ while instant gelation (namely, the aggregate size distribution vanishes for all $t > 0$) arises in the $\upsilon > 2$ case [21]. Hence, the system may undergo a gelationlike transition after a sufficiently long time (see, e.g., Refs. [24–27]). In this work, we only focus on the more interesting case of $\upsilon \leq 3/2$ case in which the aggregate size distribution infinitely evolves according to a conventional scaling regime. In the long-time limit, we obtain the asymptotic solution of $S(t)$ for the system with $\upsilon \leq 3/2$,

$$S(t) \propto \begin{cases} \exp(C_1 t) & \text{if } \upsilon = 3/2, \\ [(3-2\upsilon)t]^{1/(3-2\upsilon)} & \text{if } \upsilon < 3/2, \end{cases} \qquad (18)$$

where C_1 is a constant dependent on the details of the reaction events. Equation (18) shows that our result is in agreement with the statement of the mean aggregate size $S(t) \propto t^{1/(3-\lambda)}$ in the system with the symmetric migration rate $K(ck; cl) = K(cl; ck) = c^\lambda K(k; l)$ (see Ref. [16]).

Then we turn to determine the scaling function $\Phi(x)$. Substituting the scaling ansatz (13) into equation (2), we find the following differential equation for the scaling function $\Phi(x)$:

$$x\Phi''(x) + (2 + px^{2-\upsilon})\Phi'(x) \simeq p(\upsilon - 3)x^{1-\upsilon}\Phi(x). \qquad (19)$$

Here p is such a separation constant for the x and t dependence that satisfies $[S(t)]^{2-2\upsilon}\dot{S}(t) = Ip\int_0^\infty dx[x\Phi(x)]$. We solve equation (19) and find

$$\Phi(x) \simeq C_2 \exp(-C_3 x^{2-\upsilon}), \qquad (20)$$

where C_2 is an integration constant and $C_3 = p/(2 - \upsilon)$. Since our system obeys the mass conservation law, C_2

and p are related as $\int_0^\infty dx C_2 x^{2-\upsilon} \exp[-px^{2-\upsilon}/(2-\upsilon)] = 1$. One can choose p such that $C_2 = 1$. It is thus verified that $\Phi(x) \simeq 1$ for $x \ll 1$ and $\Phi(x) \simeq 0$ for $x \gg 1$. Here we leave p arbitrary.

Thus we know that in the migration-driven aggregation model with the symmetrical rate kernel $k(i;j) \propto (ij)^\upsilon$, the system always evolves according to a scaling law when $\upsilon \leq 3/2$. The aggregate size distribution $a_k(t)$ in the system with $\upsilon < 3/2$ approaches the conventional scaling form of equation (11) with the stretched exponential function $\Phi(x) = \exp(-C_3 x^{2-\upsilon})$ and the nonuniversal υ-dependent exponents $\tau = \upsilon - 1$, $w = (3-\upsilon)/(3-2\upsilon)$, and $z = 1/(3-2\upsilon)$. As for the borderline case of $\upsilon = 3/2$, $a_k(t)$ does not scale according to the conventional definition (11) but it satisfies the exponential-correction scaling form $a_k(t) \simeq k^{-\tau}[\exp(t)]^{-w}\Phi\{k/[\exp(t)]^z\}$ with the nonuniversal exponents $\tau = 1/2$, $w = 3C_1/2$, and $z = C_1$. The results also show that the total density of the aggregates decays as $t^{-1/(3-2\upsilon)}$ in the $\upsilon < 3/2$ case while it decreases exponentially in the special case of $\upsilon = 3/2$. Thus, each $a_k(t)$ in the system with $\upsilon \leq 3/2$ decreases with time and vanishes at $t \to \infty$.

3 The model with an asymmetrical migration rate kernel

In order to thoroughly understand the kinetics of the migration-driven aggregate growth, we also investigate another simple model with an asymmetrical power-law rate kernel $K(k;j) = 2Ik^\mu j^\nu$ ($\mu \neq \nu$). Here, the index μ is the degree of the "emigration susceptibility" of the aggregates while the index ν is the degree of the "immigration acceptability" of the aggregates. The asymmetrical model is more realistic because the residents in the smaller city may have more reasons to migrate to the larger city. It is believed that the asymmetrical migration mechanism may give the kinetic behavior different from the symmetrical one. Equation (1) is then rewritten as

$$\frac{da_k}{dt} = IM_\nu[(k+1)^\mu a_{k+1} - k^\mu a_k] + IM_\mu[(k-1)^\nu a_{k-1} - k^\nu a_k], \quad (21)$$

with the shorthand notation $M_\nu(t) \equiv \sum_{j=1}^\infty j^\nu a_j(t)$ and $M_\mu(t) \equiv \sum_{j=1}^\infty j^\mu a_j(t)$.

Following a spirit similar to the above mentioned symmetrical case, we first focus on the special case that can be solved exactly. In the case of $\mu = 0$ and $\nu = 1$, equation (21) reduces to

$$\frac{da_k}{dt} = IM_1[a_{k+1} - a_k] + IM_0[(k-1)a_{k-1} - ka_k]. \quad (22)$$

Under the unitary monodisperse initial condition, equation (22) can also be exactly solved with the help of ansatz (4). Substituting equation (4) into equation (22) we obtain

$$\frac{da}{dt} = IA, \quad \frac{dA}{dt} = -2IA(1-a), \quad (23)$$

with the the boundary condition $a(0) = 0$ and $A(0) = 1$. Equations (23) can be easily solved to yield the same exact solutions of $a(t)$ and $A(t)$ as equations (6) in the above-mentioned symmetrical model with $\upsilon = 1$. Thus we can also obtain the exact solution (7) for this special case. So, the long-time behavior of the aggregate size distribution approaches the scaling form of equation (8) and the typical aggregate size grows linearly with time. Moreover, the scaling function is also exponential, $\Phi(x) = \exp(-x)$, which is a consistent behavior for independent variable x. This is not in agreement with the statement that the scaling functions of such asymmetrical migration systems may go to zero discontinuously at a finite value in reference [16]. On the other hand, the results also show that in this case, the aggregate size distribution $a_k(t)$ continuously decays to zero at the end.

We now turn to the general case. We also make the conventional scaling ansatz (11) for the long-time behavior of $a_k(t)$ in this asymmetrical model. Moreover, it follows from the mass conservation law that both the exponent relation (12) and the modified scaling ansatz (13) also hold in this model. Multiplying equation (21) with k^2 and summing them up over all k, we obtain

$$\frac{dM_2}{dt} = 2I(M_\mu M_\nu + M_\mu M_{\nu+1} - M_{\mu+1}M_\nu). \quad (24)$$

Moreover, we also find

$$\frac{dM_0}{dt} = -Ia_1 M_\nu. \quad (25)$$

Substituting equation (13) into equations (24) and (25), we obtain two sets of evolution equations for the typical size $S(t)$ in many different cases (such as the $\tau > \nu + 2$, $\tau = \nu + 2$, $\nu + 1 < \tau < \nu + 2$, $\tau = \nu + 1$ and $\tau < \nu + 1$ cases). One can then make a detailed comparison between these resulting equations as we do in Section 2. It is found that only in the case of $\mu < \nu$ and $\tau < 1$ do the evolution behaviors of $S(t)$ derived respectively from equations (24) and (25) have the same form; furthermore, we obtain $\tau = \mu$. This indicates that the modified scaling ansatz (13) only holds in the system with $\mu < \nu$ and $\mu < 1$. The limitation $\mu < \nu$ for the scaling ansatz can also be obtained by analyzing the structure of equation (24). Inserting equation (13) into equation (24), one can readily find that if $\mu > \nu$, the long-time evolution behavior of the typical size is $dS/dt < 0$ (especially for $\nu > \tau - 1$) and the scaling ansatz is thus invalid. On the other hand, for the asymmetrical migration model with the rate kernel $K(k;l) \approx k^\mu l^\nu$ for $k \ll l$ and $K(k;l) = 0$ for $k > l$ [16], the scaling exponent τ takes the form $\tau = \mu$ only in the case of $\mu \leq \nu + 1$ and $\mu + \nu < 1$, while it has the distinct form $\tau = (1 + \mu + \nu)/2$ in the case of $\mu > \nu + 1$ and $\mu + \nu < 1$. These show that the scaling exponent τ of the aggregate size distribution depends strongly on the details of the migration rate kernel. So different migration biases may lead to distinct differences in the scaling behaviors of asymmetrical migration systems.

In this asymmetrical model, we also focus only on the kinetic scaling behavior of the system with $\mu < \nu$ and

$\mu < 1$. By inserting scaling ansatz (13) into equation (24) we obtain

$$\frac{dS}{dt} \simeq S^{\mu+\nu-1}. \quad (26)$$

Equation (26) can be straightforwardly solved to yield the asymptotic solution of $S(t)$ for $\mu + \nu < 2$,

$$S(t) \propto [(2 - \mu - \nu)t]^{1/(2-\mu-\nu)}. \quad (27)$$

This indicates that the evolution behavior of the typical size $S(t)$ for the reversible migration system is identical with that in the irreversible case (see Ref. [16]). As for the system with $\mu + \nu > 2$, the scaling ansatz (11) of the aggregate size distribution is invalid and the system will also undergo the gelationlike transition after a long time, which is similar to the kinetic behavior of the above symmetrical system with $\upsilon > 3/2$. On the other hand, it is also verified that the scaling ansatz holds only for $\mu < 1$, since the above results have exhibited that $\mu < \nu$ and $\mu + \nu < 2$.

We now determine the scaling function $\Phi(x)$ in the nongelling system with $\mu + \nu < 2$. Substituting the scaling ansatz (13) into equation (21), we find the following differential equation:

$$(IC_4 x^\nu - IC_5 x^\mu - qx)\Phi'(x) \simeq [q(2 - \mu) - IC_4(\nu - \mu)x^{\nu-1}]\Phi(x), \quad (28)$$

where $C_4 = \int_0^\infty dx\Phi(x)$, $C_5 = \int_0^\infty dx[x^{\nu-\mu}\Phi(x)]$, and q is a separation constant for the x and t dependence which satisfies $[S(t)]^{1-\mu-\nu}\dot{S}(t) = q$. It should be pointed that after some algebra, equation (28) can also be derived from the basic equation (8a) in reference [16]. Since it is difficult for us to obtain a consistent solution of $\Phi(x)$ from equation (28), we turn to analyze the important feature of $\Phi(x)$ for small values of x. For $x \ll 1$, x^μ is by far larger than both x and x^ν since we have $\mu < 1$ and $\mu < \nu$, and equation (28) then reduces to

$$\Phi^{-1}(x)\Phi'(x) \simeq [(\nu-\mu)C_4/C_5]x^{\nu-\mu-1} - [q(2-\mu)/IC_5]x^{-\mu}. \quad (29)$$

Equation (29) yields the asymptotic solution of $\Phi(x)$ for small x,

$$\Phi(x) \simeq \exp(-C_6 x^{1-\mu} + C_7 x^{\nu-\mu}), \quad (30)$$

where $C_6 = q(2 - \mu)/I(1 - \mu)C_5$ and $C_7 = C_4/C_5$. Equation (30) exhibits $\Phi(x) \simeq 1$ for $x \ll 1$, which is accord with the restriction on the scaling function $\Phi(x)$.

We then discuss the global properties of the scaling function. Leyvraz et al argued that for such an asymmetrical migration model the scaling function $\Phi(x)$ is discontinuous at a certain point x_c and $\Phi(x) \equiv 0$ for $x > x_c$ [16]. In this model, from equation (28) we also observe that the solution of the scaling function $\Phi(x)$ has singularity at the point x_c, where x_c satisfies the equation $IC_4 x_c^\nu - IC_5 x_c^\mu - qx_c = 0$. Moreover, it is not difficult to find that if $q(2 - \mu) \neq IC_4(\nu - \mu)x_c^{\nu-1}$, one may have $\Phi(x_c) = 0$. On the other hand, for our monodisperse initial model, the largest aggregate of size k at a certain time is formed only through an aggregate of size $k-1$ gaining a monomer from any other aggregates of size s ($s \le k - 1$). Thus it follows from equation (21) that $\Phi(x) = 0$ for all $x > x_c$ if $\Phi(x_c) = 0$. This is in agreement with the argument of discontinuity in reference [16]. However, under other initial condition, there is no reason for us to simply arrive at such a conclusion that $\Phi(x) = 0$ for all $x > x_c$ on the basis of $\Phi(x_c) = 0$, because an aggregate of size k can also be produced through an aggregate of size $k + 1$ losing a monomer. Moreover, it should be pointed out that discontinuity of the scaling function does not occur in each case even under the monodisperse initial condition. For example, if $\nu = 1$ and $q = IC_4$, equation (28) then yields a consistent solution $\Phi(x) \simeq \exp[-C_4 x^{1-\mu}/C_5(1 - \mu)]$ for all x, which involves the exact solution for the above case of $\mu = 0$ and $\nu = 1$.

The above discontinuity analysis can be simply argued as follows. If the scaling function has a consistent form, from equation (28) one can also derive the asymptotic solution of $\Phi(x)$ for $x \gg 1$,

$$\Phi(x) \simeq \begin{cases} x^{-(\nu-\mu)} & \text{if } \nu > 1, \\ x^{-\gamma} & \text{if } \nu = 1, \\ x^{-(2-\mu)} & \text{if } \nu < 1, \end{cases} \quad (31)$$

where $\gamma = 1 - \mu + q/(q - IC_4)$ ($q \neq IC_4$). From equation (31) one can easily find that the constant C_5 diverges in the case of $\nu > 1$. Moreover, for $\nu < 1$, it follows from $\Phi(x) \simeq x^{\mu-2}$ that the total mass $\int_0^\infty dx[x^{1-\mu}\Phi(x)]$ diverges. These will lead to an inconsistent consequence that the solution (31) of $\Phi(x)$ is in contradiction with the above scaling ansatz (13). Hence, for the asymmetrical model, the scaling function $\Phi(x)$ may be discontinuous in most cases (except for the borderline case of $\nu = 1$ and $q = IC_4$) and there exists such a point x_c that $\Phi(x)$ can be consistently set to zero for all $x > x_c$.

So far, the asymptotic scaling behavior of the asymmetrical system is acquired. The aggregate size distribution approaches a scaling law only in the case of $\mu < \nu$ and $\mu + \nu < 2$. For the aggregates A_k with $1 \ll k \ll S(t)$, its size distribution $a_k(t)$ approaches the conventional scaling form of equation (11) with the stretched exponential function (30) and some nonuniversal exponents ($\tau = \mu$, $w = (2 - \mu)/(2 - \mu - \nu)$, and $z = 1/(2 - \mu - \nu)$). The results also show that each $a_k(t)$ in this system decreases with time and vanishes finally.

4 Summary

We have introduced two distinct aggregation models in which the aggregate growth is driven by the reversible migrations between any two aggregates. Based on the mean-field assumption, we have analyzed the evolution behavior of the aggregate size distribution in the systems with and without the migration bias and found that the two systems have quite different scaling properties.

In the first model with the symmetrical rate kernel $K(k; j) \propto (kj)^\upsilon$, we found that the evolution behavior of this system depends strongly on the index υ. The aggregate size distribution approaches the conventional scaling form of equation (11) in the $\upsilon < 3/2$ case and satisfies the exponential-correction scaling law in the marginal case of $\upsilon = 3/2$; while for the $\upsilon > 3/2$ case, the system may undergo the gelationlike transition after a certain time. In particular, when $\upsilon = 1$, the aggregate size distribution approaches the well-known Smoluchovski scaling form (8). Moreover, the typical aggregate size grows as $t^{1/(3-2\upsilon)}$ in the $\upsilon < 3/2$ case while it grows exponentially in the $\upsilon = 3/2$ case.

In the second model with the asymmetrical rate kernel $K(k;j) \propto k^\mu j^\nu$ ($\mu \neq \nu$), the aggregate size distribution $a_k(t)$ also approaches the conventional scaling form (11) in the case of $\mu < \nu$ and $\mu + \nu < 2$, which reduces to the Smoluchovski scaling form (8) in the special case of $\mu = 0$ and $\nu = 1$; while in the case of $\mu+\nu > 2$ and $\mu < \nu$, the system may have a gelationlike transition; while in the case of $\mu > \nu$, infinite aggregates cannot be formed and only small aggregates can survive in the end. Moreover, the results showed that the typical aggregate size grows as $t^{1/(2-\mu-\nu)}$ in the case of $\mu < \nu$ and $\mu + \nu < 2$. Similar to the model with the symmetrical migration rate kernel, the aggregate size distribution in this asymmetrical system also decays along with time and vanishes finally. On the other hand, an interesting result in the asymmetrical case is that the exponent τ is only dependent on the index μ and independent of the index ν. So, we may conclude that the emigration of monomers (equivalently, one person) from one aggregate may play a more important role in the kinetics of the system than the migrant acceptability of another aggregate. It is natural for the growth of city populations because the migration of populations starts from the emigration of the susceptible migrants. Thus this model may be expected to provide some predictions for the distribution of city populations.

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